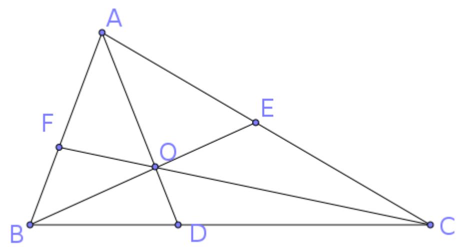
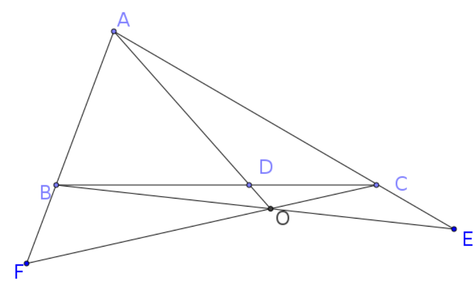
Geometrical Concepts - Part 2

1. Ceva's Theorem





If the lines joining a point O to the vertices of a triangle ABC meet the opposite sides in D, E, and F respectively, then

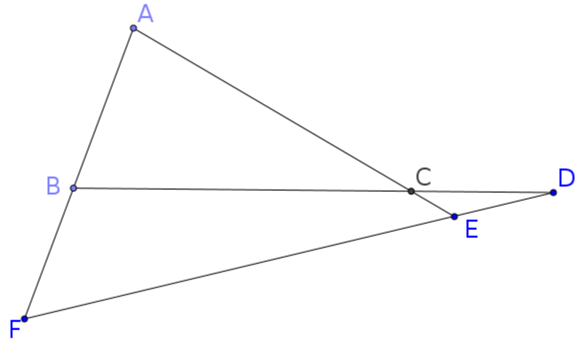
$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1.$$

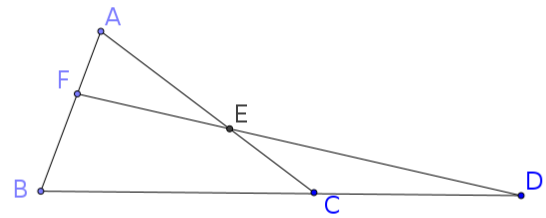
Conversely, if three points D, E, and C taken on sides BC, CA, and AB respectively of a triangle ABC are such that

$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1$$

holds, then AD, BE, and CF are concurrent.
Concurrent lines AD, BE, and CF are called Cevians.

2. Menelaus' Theorem





If three points D, E, and C taken on suitably extended sides BC, CA, and AB respectively of a triangle ABC are collinear, then

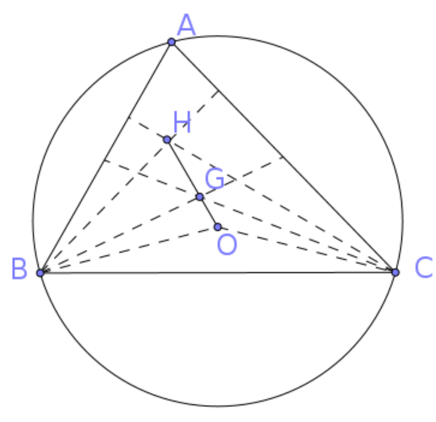
$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = -1.$$

The minus sign arises from the fact that in this theorem, all the segments are directed. For example $CA = -AC$.
Conversely, if three points D, E, and C taken on suitably extended sides BC, CA, and AB respectively of a triangle ABC are such that

$$\frac{BD}{DC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = -1$$

holds, then the points D, E, and F are collinear.

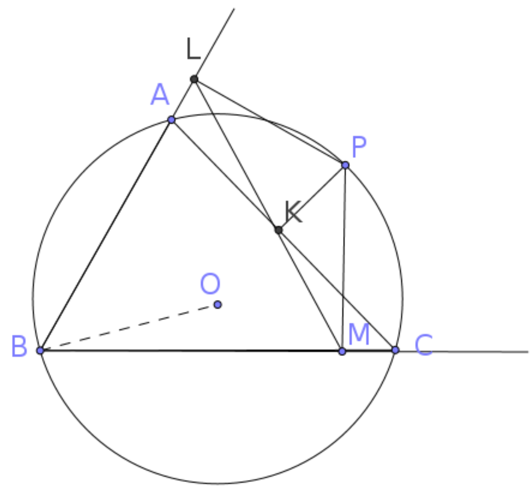
3. Euler Line



The circumcenter O, the centroid G, and the orthocenter H of an non-equilateral triangle are collinear and $GH = 2OG$. Line OGH is called the

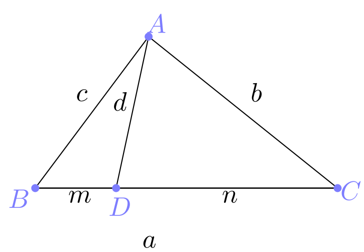
Euler line of the triangle.

4. Simson Line



The feet K, L, and M of perpendiculars from a point P on the circumcircle of a triangle ABC on suitably extended sides CA, AB, and BC respectively are collinear. The line joining K, L, and M is known as Simson line.

5. Stewart's Theorem



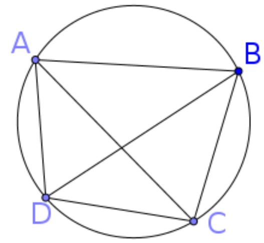
If a line drawn from vertex A of a triangle ABC to the opposite side BC meeting it at D such that $BD = m$ and $DC = n$, then

$$BC(AD^2 + mn) = m \cdot AC^2 + n \cdot AB^2.$$

If we denote the length of AD by d and the lengths of sides BC, AC, and AB by a, b, and C respectively, then the above relation is

$$a(d^2 + mn) = mb^2 + nc^2.$$

6. Ptolemy's Theorem



If $ABCD$ is a cyclic quadrilateral, then

$$AC \cdot BD = AB \cdot CD + BC \cdot AD.$$