

Geometrical Concepts - Part 1

1. Congruence of Triangles

a) Two triangles are congruent if two sides and the included angles of the first triangle are congruent to the corresponding parts of the second triangle.(S.A.S.)

b) Two triangles are congruent if two angles and the included side of the first triangle are congruent to the corresponding parts of the second triangle.(A.S.A.)

c) Two triangles are congruent if two angles and a non-included side of the first triangle are congruent to the corresponding parts of the second triangle.(A.A.S.)

d) Two triangles are congruent if the sides of the first triangle are congruent to the corresponding sides of the second triangle.(S.S.S.)

2. Similarity of Triangles

a) Two triangles are similar if any two angles of the first triangle are equal to two corresponding angles of the second triangle.

b) If two triangles have all three sides of one proportional to the three sides of the other taken in order, then the two triangles are similar.

c) Two triangles are similar if two sides of one triangle are respectively proportional to two corresponding sides of the other and if the included angles are equal.

3. Properties of Triangles

a) If two sides of a triangle are unequal, then the angle opposite to the longer side is greater than the angle opposite to the smaller side. Conversely, if two angles of a triangle are unequal, then the side opposite to

the larger angle is longer than the side opposite to the smaller angle.

- b) The sum of any two sides of a triangle is greater than the third side.
- c) The sum of the measures of the three angles of a triangle is 180° .
- d) The measure of an exterior angle of a triangle equals the sum of the measures of the two non-adjacent interior angles.
- e) The three medians of a triangle meet in a point called centroid which is situated on each median so that the measure of the segment from the vertex to the centroid is two-thirds the measure of the median.
- f) The perpendicular bisectors of the three sides of a triangle meet in a point called the circumcenter. A circle drawn with center as the circumcenter and a radius equal to the measure of the distance between the circumcenter and one of the vertices will pass through all the three vertices.
- g) The bisectors of the three angles of a triangle meet in a point called incenter. A circle drawn with center as the incenter and radius equal to the measure of the shortest distance between the incenter and one of the sides will touch tangentially all the three sides.
- h) The altitudes drawn from the three vertices of a triangle to the opposite sides meet in a point called orthocenter.
- i) If a line contains the midpoint of one side of a triangle and is parallel to a second side of the triangle, then it will bisect the third side of the triangle.
- j) The line segment whose endpoints are the midpoints of the sides of a triangle is parallel to the third side and has a measure equal to one-half of the measure of the third side.
- k) If a line is parallel to one side of a triangle, it divides the other two sides of the triangle proportionally. Conversely, if a line divides two sides of a triangle proportionally, then it is parallel to the third side.

4. Properties of Isosceles Triangles

- a) If two sides of a triangle are equal, then the angles opposite to them are also equal. Conversely, if two angles of a triangle are equal, then the sides opposite to them are also equal.

b) If the altitude from one vertex of a triangle bisects the opposite side, then the triangle is isosceles. Conversely, the altitude from the vertex formed by the equal sides of an isosceles triangle bisects the side opposite to the vertex (base).

c) If the bisector of an angle of a triangle bisects the side opposite to this angle, then the triangle is isosceles.

5. Property of Equilateral Triangles

a) An equilateral triangle is equiangular. Conversely, an equiangular triangle is equilateral.

6. Properties of Parallel Lines

a) If a pair of corresponding angles formed by a transversal of two lines are congruent, then the two lines are parallel. Conversely, if two lines are parallel, then the corresponding angles formed by a transversal of the two lines are congruent.

b) If a pair of alternate interior angles formed by a transversal of two lines are congruent, then the two lines are parallel. Conversely, if two lines are parallel, then the alternate interior angles formed by a transversal of the two lines are congruent.

c) Two lines are parallel, if they are perpendicular to the same line.

d) If a line is perpendicular to one of the two parallel lines, then it is also perpendicular to the other.

e) If a pair of consecutive interior angles formed by a transversal of two lines are supplementary, then the lines are parallel. Conversely, if two lines are parallel, then the consecutive interior angles formed by a transversal of the parallel lines are supplementary.

f) If a line segment is divided into congruent (or proportional) segments by three or more parallel lines, then any other transversal will similarly contain congruent (or proportional) segments determined by these parallel lines.

7. Properties of Quadrilaterals

- a) A quadrilateral is a parallelogram if a pair of opposite sides are both congruent and parallel.
- b) The opposite sides of a parallelogram are parallel. Conversely, if the opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram.
- c) The opposite sides of a parallelogram are congruent. Conversely, if the opposite sides of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- d) The opposite angles of a parallelogram are congruent. Conversely, if the opposite angles of a quadrilateral are congruent, then the quadrilateral is a parallelogram.
- e) Pairs of consecutive angles of a parallelogram are supplementary. Conversely, if pairs of consecutive angles of a quadrilateral are supplementary, then the quadrilateral is a parallelogram.
- f) A diagonal of a parallelogram divides the parallelogram into two congruent triangles.
- g) The diagonals of a parallelogram bisect each other. Conversely, if the diagonals of a quadrilateral bisect each other, the quadrilateral is a parallelogram.
- h) A parallelogram is a rectangle if its diagonals are congruent. Conversely, if the diagonals of a parallelogram are congruent, it is a rectangle.
- i) A rectangle is a parallelogram with four congruent angles, all right angles. Conversely, if a parallelogram has four congruent angles, then it is a rectangle.
- j) A rhombus is a parallelogram with perpendicular diagonals. Conversely, if the diagonals of a parallelogram are perpendicular to each other, the parallelogram is a rhombus.
- k) A rhombus is a quadrilateral with four congruent sides. Conversely, if a quadrilateral has all four sides congruent, then it is a rhombus.
- l) The diagonals of a rhombus bisect the angles of the rhombus.
- m) A square is a special rhombus with all four angles congruent (right angles). It has congruent diagonals which are perpendicular to each other and which also bisect each other.

- n) A trapezoid is a quadrilateral in which one pair of opposite sides are parallel.
- o) A quadrilateral is cyclic (or concyclic) if all its vertices lie on the circumference of a circle.
- p) The opposite angles of a cyclic quadrilateral are supplementary. Conversely, if the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.
- q) A quadrilateral is cyclic if one side subtends congruent angles at the two opposite vertices.

8. Properties of Circles

- a) A line perpendicular to a chord of a circle and containing the center of the circle, bisects the chord and its major and minor arcs.
- b) The perpendicular bisector of a chord of a circle contains the center of the circle.
- c) If a line is tangent to a circle, it is perpendicular to a radius at the point of tangency.
- d) A line perpendicular to the radius at a point on the circle is tangent to the circle at that point.
- e) A line perpendicular to a tangent line at the point of tangency with a circle, contains the center of the circle.
- f) The radius of a circle is perpendicular to a tangent line at the point of tangency.
- g) If a tangent line (or chord) is parallel to a secant (or chord) the arcs intercepted between these two lines are congruent.
- h) Two tangent segments to a circle from an external point are congruent.
- i) The measure of a central angle is equal to the measure of its intercepted arc.
- j) The measure of an inscribed angle equals one-half the measure of its intercepted arc.
- k) The measure of an angle whose vertex is on the circle and whose sides

are formed by a chord and a tangent line is equal to one-half the measure of the intercepted arc.

l) The measure of an angle formed by two chords intersecting inside a circle is equal to half the sum of the measures of its intercepted arc and of the arc of its vertical angle.

m) The measure of an angle formed by two secants, or a secant and a tangent line, or two tangent lines intersecting outside the circle equals one-half the difference of the measures of the intercepted arcs.

n) If two chords of a circle intersect, the product of the measures of the segments of one chord equals the product of the measures of the segments of the other chord.

o) If a tangent segment and a secant intersect outside a circle, the measure of the tangent segment is mean proportional between the measure of the secant and the measure of its external segment.

p) If two secants intersect outside the circle, the product of the measures of one secant and its external segment equals the product of the measures of the other segment and its external segment.